OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4762

Mechanics 2

Friday **27 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\ \mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

1 When a stationary firework P of mass 0.4 kg is set off, the explosion gives it an instantaneous impulse of 16 N s vertically upwards.

(i) Calculate the speed of projection of P. [2]

While travelling vertically upwards at $32\,\mathrm{m\,s^{-1}}$, P collides directly with another firework Q, of mass 0.6 kg, that is moving directly downwards with speed $u\,\mathrm{m\,s^{-1}}$, as shown in Fig. 1. The coefficient of restitution in the collision is 0.1 and Q has a speed of $4\,\mathrm{m\,s^{-1}}$ vertically *upwards* immediately after the collision.

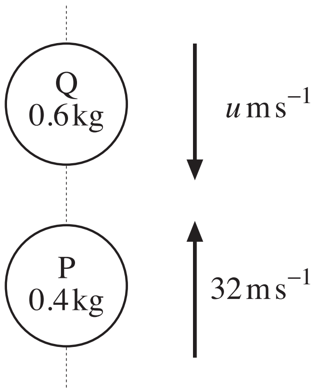


Fig. 1

(ii) Show that $u = 18$ and calculate the speed and direction of motion of P immediately after the collision. [7]

Another firework of mass 0.5 kg has a velocity of $(-3.6\mathbf{i} + 5.2\mathbf{j})\,\mathrm{m\,s^{-1}}$, where **i** and **j** are horizontal and vertical unit vectors, respectively. This firework explodes into two parts, C and D. Part C has mass 0.2 kg and velocity $(3\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ immediately after the explosion.

(iii) Calculate the velocity of D immediately after the explosion in the form $a\mathbf{i} + b\mathbf{j}$. Show that C and D move off at 90° to one another. [8]

2 A uniform beam, AB, is 6 m long and has a weight of 240 N.

Initially, the beam is in equilibrium on two supports at C and D, as shown in Fig. 2.1. The beam is horizontal.

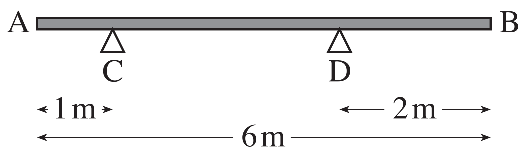


Fig. 2.1

(i) Calculate the forces acting on the beam from the supports at C and D. [4]

A workman tries to move the beam by applying a force T N at A at 40° to the beam, as shown in Fig. 2.2. The beam remains in horizontal equilibrium but the reaction of support C on the beam is zero.

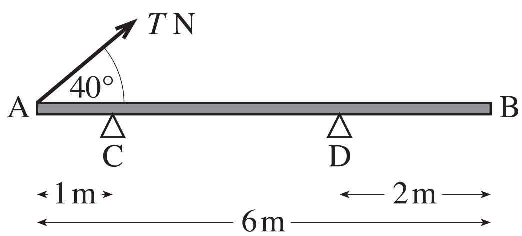


Fig. 2.2

(ii) (*A*) Calculate the value of T. [4]

(*B*) Explain why the support at D cannot be smooth. [1]

The beam is now supported by a light rope attached to the beam at A, with B on rough, horizontal ground. The rope is at 90° to the beam and the beam is at 30° to the horizontal, as shown in Fig. 2.3. The tension in the rope is P N. The beam is in equilibrium on the point of sliding.

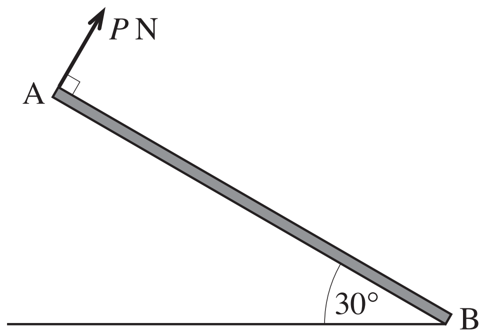


Fig. 2.3

(iii) (*A*) Show that $P = 60\sqrt{3}$ and hence, or otherwise, find the frictional force between the beam and the ground. [5]

(*B*) Calculate the coefficient of friction between the beam and the ground. [5]

 [Turn over

3 **(a)** A uniform lamina made from rectangular parts is shown in Fig. 3.1. All the dimensions are centimetres. All coordinates are referred to the axes shown in Fig. 3.1.

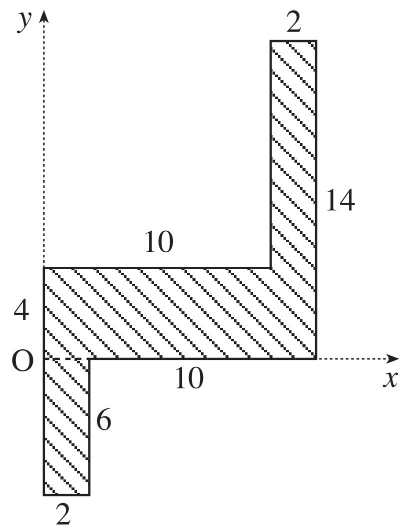


Fig. 3.1

(i) Show that the x-coordinate of the centre of mass of the lamina is 6.5 and find the y-coordinate. [5]

A square of side 2 cm is to be cut from the lamina. The sides of the square are to be parallel to the coordinate axes and the centre of the square is to be chosen so that the x-coordinate of the centre of mass of the new shape is 6.4.

(ii) Calculate the x-coordinate of the centre of the square to be removed. [3]

The y-coordinate of the centre of the square to be removed is now chosen so that the y-coordinate of the centre of mass of the final shape is as large as possible.

(iii) Calculate the y-coordinate of the centre of mass of the lamina with the square removed, giving your answer correct to three significant figures. [3]

(b) Fig. 3.2 shows a framework made from light rods of length 2 m freely pin-jointed at A, B, C, D and E. The framework is in a vertical plane and is supported at A and C. There are loads of 120 N at B and at E. The force on the framework due to the support at A is R N.

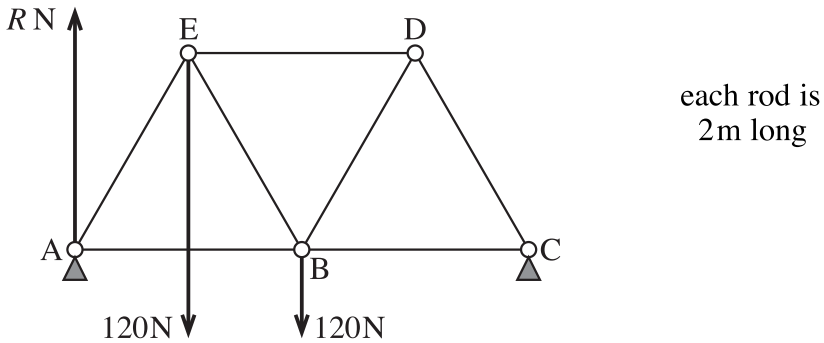


Fig. 3.2

(i) Show that $R = 150$. [2]

(ii) Draw a diagram showing all the forces acting at the points A, B, D and E, including the forces internal to the rods.

Calculate the internal forces in rods AE and EB, and determine whether each is a tension or a thrust. [You may leave your answers in surd form.] [6]

(iii) Without any further calculation of the forces in the rods, explain briefly how you can tell that rod ED is in thrust. [1]

[Question 4 is printed overleaf.]

4 A block of mass 20 kg is pulled by a light, horizontal string over a rough, horizontal plane. During 6 seconds, the work done against resistances is 510 J and the speed of the block increases from $5\,\mathrm{m\,s^{-1}}$ to $8\,\mathrm{m\,s^{-1}}$.

(i) Calculate the power of the pulling force. [4]

The block is now put on a rough plane that is at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. The frictional resistance to sliding is $11g$ N. A light string parallel to the plane is connected to the block. The string passes over a smooth pulley and is connected to a freely hanging sphere of mass m kg, as shown in Fig. 4.

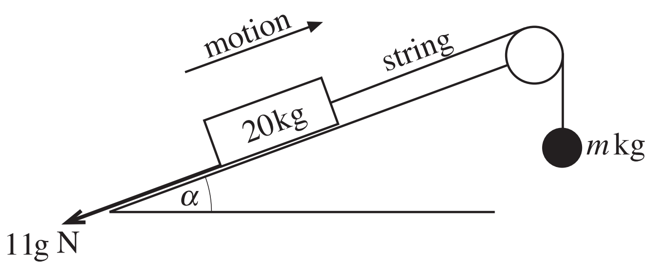


Fig. 4

In parts **(ii)** and **(iii)**, the sphere is pulled downwards and then released when travelling at a speed of $4\,\mathrm{m\,s^{-1}}$ vertically downwards. The block never reaches the pulley.

(ii) Suppose that $m = 5$ and that after the sphere is released the block moves x m up the plane before coming to rest.

(A) Find an expression in terms of x for the change in gravitational potential energy of the system, stating whether this is a gain or a loss. [4]

(B) Find an expression in terms of x for the work done against friction. [1]

(C) Making use of your answers to parts (A) and (B), find the value of x. [3]

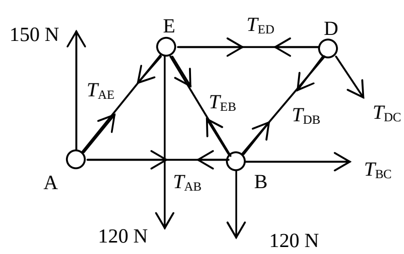
(iii) Suppose instead that $m = 15$. Calculate the speed of the sphere when it has fallen a distance 0.5 m from its point of release. [4]

Mark Scheme 4762
January 2006

Q 1		mark		Sub
(i)	$16 = 0.4v$ so 40 m s $^{-1}$	M1 A1	Use of $I = \Delta mv$	2
(ii)	PCLM ↑ + ve $0.4\times 32 - 0.6u = 0.4v_P + 0.6\times 4$ NEL ↑ +ve $\frac{4 - v_P}{-u - 32} = -0.1$ Solving u = 18 $v_P = -1$ so 1 m s $^{-1}$ downwards	M1 A1 M1 A1 E1 A1 A1	Use of PCLM Any form Use of NEL. Allow sign errors. Any form Must be obtained from a pair of correct equations. If given u = 18 used then v_P = -1 must be obtained from 1 equation and both values tested in the second equation cao. Accept use of given u = 18 cao	7
(iii)	Considering the momenta involved $0.5\begin{pmatrix}-3.6\\5.2\end{pmatrix} = 0.2\begin{pmatrix}3\\4\end{pmatrix} + 0.3\mathbf{v}_D$ $\mathbf{v}_D = \begin{pmatrix}-8\\6\end{pmatrix}$ so $a = -8$ and $b = 6$ Gradients of the lines are $\frac{4}{3}$ and $\frac{6}{-8}$ Since $\frac{4}{3}\times\frac{6}{-8} = -1$, they are at 90°	M1 B1 B1 A1 A1 A1 M1 E1	PCLM applied. May be implied. LHS momentum of C correct Complete equation. Accept sign error. cao cao Any method for the angle Clearly shown	8
				17

Q 2		mark		Sub
(i)	Moments about C $240 \times 2 = 3R_D$ $R_D = 160$ so 160 N Resolve vertically $R_C + R_D = 240$ $R_C = 80$ so 80 N	M1 A1 M1 F1	Moments about C or equivalent. Allow 1 force omitted Resolve vertically or moments about D or equivalent. All forces present. FT from **thei**r R_D only	4
(ii) (A)	Moments about D $240 \times 1 = 4T \sin 40$ T = 93.343… so 93.3 N (3 s. f.)	M1 M1 A1 A1	Moments about D or equivalent Attempt at resolution for RHS RHS correct	4
(ii) (B)	In equilibrium so horizontal force needed to balance cpt of T. This must be friction and cannot be at C.		Need reference to horizontal force that must come from friction at D.	1
(iii) (A)	Moments about B $3 \times 240 \times \cos 30 = 6P$ $P = 60\sqrt{3}$ (103.92…..) P inclined at 30° to vertical Resolve horizontally. Friction force F $F = P \sin 30$ so $F = 30\sqrt{3}$ (51.961…)	M1 E1 B1 M1 A1	All terms present, no extras. Any resolution required attempted. Accept decimal equivalent Seen or equivalent or implied in (iii) (A) or (B). Resolve horizontally. Any resolution required attempted Any form	5

(iii) (B)	Resolve vertically. Normal reaction *R* $P\cos 30 + R = 240$	M1 A1	Resolve vertically. All terms present.and resolution attempted	
	Using $F = \mu R$ $\mu = \dfrac{30\sqrt{3}}{240 - 60\sqrt{3} \times \dfrac{\sqrt{3}}{2}}$	M1 A1	Substitute **their expressions** for *F* and *R*	
	$= \dfrac{30\sqrt{3}}{240-90} = \dfrac{\sqrt{3}}{5}$ = 0.34641 so 0.346 (3 s. f.)	A1	cao. Any form. Accept 2 s. f. or better	5
				19

Q 3		mark		Sub
(a)(i)	$80\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=48\begin{pmatrix}6\\2\end{pmatrix}+12\begin{pmatrix}1\\-3\end{pmatrix}+20\begin{pmatrix}11\\9\end{pmatrix}$	M1	Correct method for c.m.	
	$80\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}520\\240\end{pmatrix}$	B1	Total mass correct	
		B1	One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$\bar{x}=6.5$	E1		
	$\bar{y}=3$	A1	cao	5
(ii)	Consider *x* coordinate			
	$520=76\times6.4+4x$	M1	Using additive principle o. e. on *x* cpts	
		B1	Areas correct. Allow FT from masses from (i)	
	so $x=8.4$	A1	cao	3
(iii)	*y* coordinate is 1 so we need	B1	Position of centre of square	
	$240=76\bar{y}+4\times1$ and $\bar{y}=3.10526...$	M1		
	so 3.11 (3 s. f.)	A1	cao	3
(b)(i)	Moments about C			
	$4R=120\times3+120\times2$	M1	Moments equation. All terms present	
	so 4 *R* = 600 and *R* = 150	E1		2
(ii)		B1		
	A↑ $150+T_{AE}\cos30=0$	M1	Equilibrium at a pin-joint	
	$T_{AE}=-100\sqrt{3}$ so $100\sqrt{3}$ N (C)	A1	Any form. Sign correct. Neglect (C)	
	E↓ $120+T_{AE}\cos30+T_{EB}\cos30=0$	M1	Equilibrium at E, all terms present	
	$T_{EB}=20\sqrt{3}$ so $20\sqrt{3}$ N (T)	F1	Any form. Sign follows working. Neglect (T).	
		F1	T/C consistent with answers	

				6
(iii)	Consider $\rightarrow$ at E, using (ii) gives ED as thrust	E1	Clearly explained. Accept 'thrust' correctly deduced from wrong answers to (ii).	1
				20

Q 4		mark		Sub
(i)	$\frac{0.5\times 20\times 8^2 - 0.5\times 20\times 5^2 + 510}{6}$	M1	Use of P = WD/t	
		B1	ΔKE. Accept ± 390 soi	
		A1	All correct including signs	
	= 150 W	A1		4
(ii) (A)	$20g\times\frac{3}{5}x - 5gx$	M1	Use of mgh on both terms	
		B1	Either term (neglecting signs)	
	$7gx$ ($68.6x$) gain	A1	$\pm 7gx$ in any form.	
		A1	cao	4
(B)	$11gx$	B1		1
(C)	$0.5\times 25\times 4^2 = 7gx + 11gx = 18gx$	M1	Use of work-energy equation. Allow 1 RHS term omitted.	
		B1	KE term correct	
	x = 1.13378… so 1.13 m (3 s. f.)	A1	cao. Except follow wrong sign for $7gx$ only.	3
(iii)	**either**			
	$0.5\times 35\times v^2 - 0.5\times 35\times 16$	M1	Use of work-energy. KE, GPE and WD against friction terms present.	
	$= 15g\times 0.5 - 11g\times 0.5 - 12g\times 0.5$	B1	ΔGPE correct inc sign (1.5g J loss)	
		A1	All correct	
	$v^2 = 13.76$ so v = 3.70944… so 3.71 m s^{-1} (3 s. f.)	A1	cao	
	or			
	$15g - T = 15a$ $\quad T - 12g - 11g = 20a$	M1	N2L in 1 or 2 equations. All terms present	
	so $a = -2.24$	A1	cao	
	$v^2 = 4^2 + 2\times(-2.24)\times 0.5$	M1	Use of appropriate (sequence of) *uvast*	
	so 3.71 m s^{-1} (3 s. f.)	A1	cao	4
				16

4762: Mechanics 2 (Written Examination)

General Comments

The majority of candidates found this paper to be quite accessible with many able to obtain at least some credit on some part of every question. There was some evidence that a few candidates found the paper long. As in previous sessions the standard of presentation was high and some excellent work was seen from a large number of candidates. However, many candidates do not seem to understand the value of a diagram both in assisting them to a solution and in clarifying working to an examiner. Questions that required candidates to show a given answer or explain an answer posed problems to a significant number of candidates with many not appreciating the need to explain fully and clearly the principles or processes involved.

Comments on Individual Questions

1 **Impulse and Momentum**

Overall, this question posed few problems to the vast majority of candidates.

i) Almost all candidates could obtain full credit for this part.

ii) Sign errors were common in this part particularly when using Newton's experimental law. Those candidates who drew a diagram and then constructed equations consistent with it were inevitably more successful than those candidates who either did not draw a diagram or who drew one and then ignored it. Candidates who did not draw a diagram usually failed to explain their sign convention when showing that u = 18 and also omitted the direction when describing the motion of P.

iii) It was pleasing to see many complete and accurate answers to this part of the question. The majority of candidates understood the need to look at the motion in 2 dimensions and only a minority failed to establish that the velocity of D was $-8\mathbf{i} + 6\mathbf{j}$. Showing that C and D move off at right angles to each other was less well done. Most candidates calculated the directions of the two particles as angles and showed that these added to give 90° but then failed to show clearly why the angle **between** the parts was 90°. The more able candidates used either the scalar product or showed that the product of the gradients was –1.

2 **Resolving and moments**

This question was well done by many candidates

i) Almost all the candidates obtained full marks for this part.

ii) Most candidates realised that they had to take moments and could establish the correct value for T. A small minority made the problem more complicated by taking moments about either the centre of mass or about B and were then unable to proceed further. Others assumed that the reactions at C and D were the same as in part i) and tried to calculate T by resolving only. Few candidates could explain clearly why the support at D could not be smooth. To maintain horizontal equilibrium, a force at D was required to counteract the horizontal component of T. This could only come from the friction at D since the reaction at C was zero. Many candidates merely said that the beam would slide or slip if there was no friction.

iii) Most candidates could establish $P = 60\sqrt{3}$ but could not resolve correctly to obtain the frictional force. Most realised that the normal reaction at B needed to be calculated in order to find the coefficient of friction but some creative algebra and arithmetic was seen by those candidates who were determined to show that $\mu = \tan 30$.

3 Centres of mass

Many candidates scored well on both parts of this question. Part a) was generally attempted more successfully than part b)

a) i) The majority of candidates scored highly in this part.

ii) This posed few problems to the majority.

iii) This part caused difficulty to a minority of candidates; these did not appreciate that the y co-ordinate of the square to be removed had to be where $y = 1$. Some tried to take the square from the part below the x axis, others misinterpreted the question and took the maximum value possible for the centre of the square to be removed.

b) i) Candidates were almost invariably successful in establishing the given result.

ii) On the whole, the standard of diagrams drawn was poor. Many were less than helpful to the candidates as they had inadequately labelled forces, omitted forces or had more than one force with the same label. The directions of the internal forces were not made clear. Some candidates drew a diagram and then ignored it when calculating the internal forces. This gave rise to sign errors and inconsistencies between equations. A very small number of candidates penalised themselves in terms of time by calculating all of the internal forces, not just those that were requested. Those candidates who only drew small and separate diagrams for the forces at each pin joint were not usually as successful as those who drew a complete diagram. The directions of the forces were not always consistent between diagrams.

iii) This part was not well done by the majority of candidates. Many did not understand the need to look at the horizontal equilibrium and the direction of the calculated forces. Others stated without justification that the system would collapse.

4 Work and energy

This question was not as well attempted as the previous questions. It caused difficulties for a significant minority of candidates who seemed unsure about the principles involved.

i) While most candidates understood that power was the rate of doing work, many assumed that the total work done was 510 J. A very small minority understood that work was being done by the tension in the string. However, instead of using the change in kinetic energy as a measure of this, overcomplicated matters by attempting to use Newton’s second law and the constant acceleration formulae to find the acceleration and distance travelled. Very few attempting this were successful.

ii) A) Many candidates successfully completed this part but a minority forgot one or other of the two masses involved. The main errors occurred when candidates treated the mass m as if it lay on the inclined plane.

B) This caused no problem to the vast majority.

C) Again, few problems were encountered by those who had successfully completed A and B. The main error was in using the mass of only one of the objects in the calculation of the kinetic energy instead of the combined mass of 25 kg.

iii) Many of the candidates completed this part successfully. Omitting one of the kinetic energy terms or the gravitational potential energy terms was the main cause of error for others. Candidates who used Newton’s second law and the constant acceleration formulae did not score as highly on the whole as those who used work-energy methods